Table of Laplace Transforms

	Function	Laplace Transform		Function	Laplace Transform		Function	Laplace Transform
	$f(t)$	$F(s)=\int_0^\infty e^{-st}f(t)\,\mathrm{d}t$		$f(t)$	$F(s)=\int_0^\infty e^{-st}f(t)\,\mathrm{d}t$		$f(t)$	$F(s)=\int_0^\infty e^{-st}f(t)\,\mathrm{d}t$
1	1	$\frac{1}{s}$	14	$\frac{2}{t}\,(1-\cos(\omega t))$	$\log\left(\frac{s^2+\omega^2}{s^2}\right)$	27	$\frac{1}{\sqrt{t}}\,\cos\left(2\sqrt{at}\right)$	$\sqrt{\frac{\pi}{s}}\,e^{-(a/s)}$
2	t^n	$\frac{n!}{s^{n+1}},\ n=0,1,2,3,\ldots$	15	$\frac{2}{t}\,\sinh(\omega t)$	$\log\left(\frac{s+\omega}{s-\omega}\right)$	28	$\frac{1}{\sqrt{t}}\,\sin\left(2\sqrt{at}\right)$	$\sqrt{\frac{\pi}{s}}\,e^{-(a/s)}\mathrm{erfi}\left(\sqrt{\frac{a}{s}}\right)$
3	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2}s^{-3/2}$	16	$\frac{2}{t}\,(1-\cosh(\omega t))$	$\log\left(\frac{s^2-\omega^2}{s^2}\right)$	29	$J_0(at)$	$\frac{1}{\sqrt{a^2+s^2}}$
4	e^{at}	$\frac{1}{s-a}$	17	$\sin(\omega t)\sinh(at)$	$\frac{2as\omega}{(s^2+\omega^2-a^2)^2+4a^2\omega^2}$	30	$J_0\left(2\sqrt{at}\right)$	$\frac{1}{s}\,e^{-(a/s)}$
5	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	18	$\cos(\omega t)\sinh(at)$	$\frac{a\,(s^2-\omega^2-a^2)}{(s^2+\omega^2-a^2)^2+4a^2\omega^2}$	31	$I_0(at)$	$\frac{1}{\sqrt{s^2-a^2}}$
6	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	19	$\sin(\omega t)\cosh(at)$	$\frac{\omega\,(s^2+\omega^2+a^2)}{(s^2+\omega^2-a^2)^2+4a^2\omega^2}$	32	$I_0\left(2\sqrt{at}\right)$	$\frac{1}{s}\,e^{(a/s)}$
7	$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$	20	$\cos(\omega t)\cosh(at)$	$\frac{s\,(s^2+\omega^2-a^2)}{(s^2+\omega^2-a^2)^2+4a^2\omega^2}$	33	$Y_0(at)$	$-\frac{2}{\pi}\frac{\mathrm{arsinh}\left(\frac{s}{a}\right)}{\sqrt{a^2+s^2}}$
8	$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$	21	$\delta(t-a)$	$e^{-as},\ a\geq 0$	34	$K_0(at)$	$\frac{\arccos\left(\frac{s}{\lvert a\rvert}\right)}{\sqrt{a^2-s^2}}$
9	$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$	22	$H(t-a)$	$\frac{1}{s}\,e^{-as},\ a\geq 0$	35	$J_n(at)$	$\frac{\left(\sqrt{a^2+s^2}-s\right)^n}{a^n\sqrt{a^2+s^2}}$
10	$e^{at}\cos(\omega t)$	$\frac{s-a}{(s-a)^2+\omega^2}$	23	$\frac{1}{\sqrt{t}}\,e^{-(a^2/t)}$	$\sqrt{\frac{\pi}{s}}\,e^{-2\sqrt{a^2s}}$	36	$I_n(at)$	$\frac{\left(s-\sqrt{s^2-a^2}\right)^n}{a^n\sqrt{s^2-a^2}}$
11	$t\sin(\omega t)$	$\frac{2s\omega}{\left(s^2+\omega^2\right)^2}$	24	$\frac{1}{t\sqrt{t}}\,e^{-(a^2/t)}$	$\sqrt{\frac{\pi}{a^2}}\,e^{-2\sqrt{a^2s}}$	37	$\sin(\omega t)-at\cos(\omega t)$	$\frac{\omega(s^2+\omega^2)-a(s^2-\omega^2)}{\left(s^2+\omega^2\right)^2}$
12	$t\cos(\omega t)$	$\frac{s^2-\omega^2}{\left(s^2+\omega^2\right)^2}$	25	$\mathrm{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{1}{s}e^{-\sqrt{a^2s}}$	38	$\sin(\omega t)+at\cos(\omega t)$	$\frac{\omega(s^2+\omega^2)+a(s^2-\omega^2)}{\left(s^2+\omega^2\right)^2}$
13	$\frac{1}{t}\,\sin(\omega t)$	$\arctan\left(\frac{\omega}{s}\right)$	26	$\mathrm{erf}\left(\sqrt{at}\right)$	$\frac{\sqrt{a}}{s\sqrt{a+s}}$	39	$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$

- $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$, the error function, $\mathrm{erfc}(z) = 1 - \mathrm{erf}(z)$, the complementary error function and $\mathrm{erfi}(z) = \mathrm{erf}(iz)/i$, the imaginary error function.
- $\delta(x)$ is the Dirac delta function.
- $H(x)$ is the Heaviside unit step function.
- $J_n(z)$, $I_n(z)$, $Y_n(z)$ and $K_n(z)$ are Bessel functions of the first kind, modified first kind, second kind and modified second kind respectively.

Operational Formulas for Laplace Transforms

	$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,\mathrm{d}t$	$\mathcal{L}^{-1}\{F(s)\} = f(t)$
Linearity	$\mathcal{L}\{a\,f(t) + b\,g(t)\} = a\,F(s) + b\,G(s), \quad \text{if } a, b \in \mathcal{R}$	$\mathcal{L}^{-1}\{a\,F(s) + b\,G(s)\} = a\,f(t) + b\,g(t)$
Scaling	$\mathcal{L}\left\{f\left(\frac{t}{a}\right)\right\} = a\,F(as)$	$\mathcal{L}^{-1}\{F(as)\} = \frac{1}{a} f(t/a)$
First Shift	$\mathcal{L}\{e^{at}\,f(t)\} = F(s-a)$	$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}\{F(s)\} = e^{at} f(t)$
Second Shift	$\mathcal{L}\{f(t-a)\,H(t-a)\} = e^{-as}\,F(s)$, if $a > 0$.	$\mathcal{L}^{-1}\{e^{-as}\,F(s)\} = f(t-a)\,H(t-a)$
Convolution	$\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\left\{\int_0^t f(u)\,g(t-u)\,du\right\} = F(s)\,G(s)$	$\mathcal{L}^{-1}\{F(s)\,G(s)\} = \int_0^t f(u)\,g(t-u)\,du$
Mult. by t	$\mathcal{L}\{t\,f(t)\} = -F'(s) \qquad \text{and } \mathcal{L}\{t^n\,f(t)\} = (-1)^n\,F^{(n)}(s)$	
Derivatives	$\mathcal{L}\{f'(t)\} = s\,F(s) - f(0) \qquad \text{and } \mathcal{L}\{f''(t)\} = s^2\,F(s) - s\,f(0) - f'(0)$ and $\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n\,F(s) - s^{n-1}\,f(0) - s^{n-2}\,f'(0) - \ldots\ldots - s\,f^{(n-2)}(0) - f^{(n-1)}(0)$	
Integrals	$\mathcal{L}\left\{\int_0^t f(u)du\right\} = \frac{1}{s}\,F(s) \qquad \text{and } \mathcal{L}\left\{\int_a^t f(u)du\right\} = \frac{1}{s}\,F(s) - \frac{1}{s}\int_0^a f(u)du$	
Div. by t	$\mathcal{L}\left\{\frac{1}{t}\,f(t)\right\} = \int_s^\infty F(u)du$	
Periodic	$f(t+\tau) = f(t), \quad \mathcal{L}\{f(t)\} = \frac{1}{1-e^{-\tau s}} \int_0^\tau e^{-st} f(t)dt$	

Differentiation and integration rules

$\frac{d}{dx}(u \cdot v) = u'v + uv'$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$\frac{d}{dx}(f(g)) = f'(g)\, g'(x)$

$\int f(g)\, g'(x)\, \mathrm{d}x = \int f(u)\, \mathrm{d}u$

$\int u \frac{dv}{dx}\, \mathrm{d}x = uv - \int v \frac{du}{dx}\, \mathrm{d}x$

Trigonometry

π radians equals 180°, 1° equals $\frac{\pi}{180}$ radians

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Hyperbolic

$\sinh x = \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})$, $\cosh x = \frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x})$

$\tanh x = \frac{\sinh x}{\cosh x}$, $\coth x = \frac{\cosh x}{\sinh x}$

$\mathrm{csch} x = \frac{1}{\sinh x}$, $\mathrm{sech} x = \frac{1}{\cosh x}$

$\cosh^2 x - \sinh^2 x = 1$, $\tanh^2 x + \mathrm{sech}^2 x = 1$

$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1$
$= 2\sinh^2 x + 1$

$\cosh x \pm \sinh x = \mathrm{e}^{\pm x}$

Logarithms and exponents

$a^{n+m} = a^n a^m$, $(a^m)^n = a^{mn}$

$a^m / a^n = a^{m-n}$, $(ab)^n = a^n b^n$

$\log(xy) = \log x + \log y$, $\log(x^n) = n \log x$

$\log\left(\frac{x}{y}\right) = \log x - \log y$, $\log_b a = \frac{\log a}{\log b}$

Vectors

Length: $|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

$\mathbf{a} \times \mathbf{b} = (a_2 b_3 - a_3 b_2)\mathbf{i} + (a_3 b_1 - a_1 b_3)\mathbf{j}$
$+ (a_1 b_2 - a_2 b_1)\mathbf{k}$

Line through point (x_0, y_0, z_0) parallel to (a, b, c):
$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Plane with normal (a, b, c) is: $ax + by + cz = d$

$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{b} \cdot \mathbf{c}$

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Multivariable calculus

Gradient: $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

Divergence: $\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

Curl: $\nabla \times \mathbf{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\mathbf{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\mathbf{j}$
$+ \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\mathbf{k}$

Directional Derivative: $\left.\frac{\mathrm{d}f}{\mathrm{d}s}\right|_{\hat{\mathbf{a}}} = \nabla f \cdot \hat{\mathbf{a}}$

Area element: $\mathrm{d}A = \mathrm{d}x\, \mathrm{d}y = r\, \mathrm{d}r\, \mathrm{d}\theta$

Cyl. polar coords. (s, θ, z): $\mathrm{d}V = s\, \mathrm{d}s\, \mathrm{d}\theta\, \mathrm{d}z$

Sph. polar coords. (r, θ, ϕ): $\mathrm{d}V = r^2 \sin\phi\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}\phi$

Surface: $F(x, y, z) = 0$ Normal: ∇F

Surface: $\mathbf{r} = \mathbf{r}(s, t)$ Normal: $\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t}$

Curve: $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$
Tangent: $\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}\mathbf{i} + \frac{\mathrm{d}y}{\mathrm{d}t}\mathbf{j} + \frac{\mathrm{d}z}{\mathrm{d}t}\mathbf{k}$

Arc-length: $\frac{\mathrm{d}s}{\mathrm{d}t} = |\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}|$

Stokes' theorem: $\iint_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}}\, \mathrm{d}S = \oint_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$
where C bounds S.

Gauss' Div. Thm.: $\iiint_V \nabla \cdot \mathbf{F}\, \mathrm{d}V = \iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, \mathrm{d}S$,
surface S bounds V.

Green's Thm.: $\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) \mathrm{d}x\, \mathrm{d}y$
$= \oint_C (f\, \mathrm{d}x + g\, \mathrm{d}y)$, C bounds region R.

Complex numbers

$z = x + iy$, $\bar{z} = x - iy$

$i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$\mathrm{Re}(z) = x$, $\mathrm{Im}(z) = y$, $|z| = \sqrt{x^2 + y^2}$

$\mathrm{e}^{\pm i\theta} = \cos\theta \pm i \sin\theta$

If $z = re^{i\theta}$ then $r = |z|$ and $\theta = \arg z$

$|z^n| = |z|^n$ $\arg(z^n) = n \arg(z) \pm 2k\pi$

$\sinh(iz) = i \sin z$ $\sin(iz) = i \sinh z$

$\cosh(iz) = \cos z$ $\cos(iz) = \cosh z$

Table of Integrals

	$f(x)$	$\int f(x)\,\mathrm{d}x$		$f(x)$	$\int f(x)\,\mathrm{d}x$		$f(x)$	$\int f(x)\,\mathrm{d}x$
1	x^n	$\dfrac{x^{n+1}}{n+1},\ n \neq -1$	2	$(ax+b)^n$	$\dfrac{(ax+b)^{n+1}}{a(n+1)},\ n \neq -1$	3	$\dfrac{1}{x}$	$\log\lvert x\rvert$
4	$\dfrac{1}{ax+b}$	$\frac{1}{a}\log\lvert ax+b\rvert$	5	e^{kx}	$\frac{1}{k}\mathrm{e}^{kx}$	6	$(ax+b)\mathrm{e}^{kx}$	$\left(\frac{1}{k}(ax+b) - \frac{a}{k^2}\right)\mathrm{e}^{kx}$
7	$\sin(\omega x)$	$-\frac{1}{\omega}\cos(\omega x)$	8	$\cos(\omega x)$	$\frac{1}{\omega}\sin(\omega x)$	9	$\tan(\omega x)$	$-\frac{1}{\omega}\log\lvert\cos(\omega x)\rvert$
10	$\cot(\omega x)$	$\frac{1}{\omega}\log\lvert\sin(\omega x)\rvert$	11	$\sec(\omega x)$	$\frac{1}{\omega}\log\lvert\sec(\omega x)+\tan(\omega x)\rvert$	12	$\csc(\omega x)$	$\frac{1}{\omega}\log\lvert\csc(\omega x)-\cot(\omega x)\rvert$
13	$\sec^2(\omega x)$	$\frac{1}{\omega}\tan(\omega x)$	14	$\csc^2(\omega x)$	$-\frac{1}{\omega}\cot(\omega x)$	15	$\sec(\omega x)\tan(\omega x)$	$\frac{1}{\omega}\sec(\omega x)$
16	$\csc(\omega x)\cot(\omega x)$	$-\frac{1}{\omega}\csc(\omega x)$	17	$\sinh(\omega x)$	$\frac{1}{\omega}\cosh(\omega x)$	18	$\cosh(\omega x)$	$\frac{1}{\omega}\sinh(\omega x)$
19	$\tanh(\omega x)$	$\frac{1}{\omega}\log(\cosh(\omega x))$	20	$\coth(\omega x)$	$\frac{1}{\omega}\log\lvert\sinh(\omega x)\rvert$	21	$\mathrm{sech}(\omega x)$	$\frac{2}{\omega}\arctan\left(\tanh\left(\frac{\omega x}{2}\right)\right)$
22	$\mathrm{csch}(\omega x)$	$\frac{1}{\omega}\log\left(\tanh\left(\frac{\omega x}{2}\right)\right)$	23	$\dfrac{a}{a^2+x^2}$	$\arctan\left(\frac{x}{a}\right)$	24	$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\frac{x}{a}\right)$
25	$\dfrac{1}{\sqrt{x^2-a^2}}$	$\mathrm{arcosh}\left(\frac{x}{a}\right)$	26	$\dfrac{1}{\sqrt{a^2+x^2}}$	$\mathrm{arsinh}\left(\frac{x}{a}\right)$	27	$\sin(\omega x)\cos^n(\omega x)$	$\frac{-\cos^{n+1}(\omega x)}{\omega(n+1)},\ n \neq -1$
28	$\cos(\omega x)\sin^n(\omega x)$	$\frac{\sin^{n+1}(\omega x)}{\omega(n+1)},\ n \neq -1$	29	$\sin^2(\omega x)$	$\frac{x}{2} - \frac{1}{4\omega}\sin(2\omega x)$	30	$\cos^2(\omega x)$	$\frac{x}{2} + \frac{1}{4\omega}\sin(2\omega x)$

	$f(x)$	$\int f(x)\,\mathrm{d}x$		$f(x)$	$\int f(x)\,\mathrm{d}x$
31	$(ax+b)\sin(\omega x)$	$\frac{a}{\omega^2}\sin(\omega x) - \frac{1}{\omega}(ax+b)\cos(\omega x)$	32	$(ax+b)\cos(\omega x)$	$\frac{a}{\omega^2}\cos(\omega x) + \frac{1}{\omega}(ax+b)\sin(\omega x)$
33	$\arcsin(kx)$	$x\arcsin(kx) + \frac{1}{k}\sqrt{1-k^2x^2}$	34	$\arccos(kx)$	$x\arccos(kx) - \frac{1}{k}\sqrt{1-k^2x^2}$
35	$\arctan(kx)$	$x\arctan(kx) - \frac{1}{2k}\log\left(1+k^2x^2\right)$	36	$\dfrac{a}{a^2-x^2}$	$\begin{cases}\mathrm{artanh}\left(\frac{x}{a}\right) = \frac{1}{2}\log\left(\frac{a+x}{a-x}\right), & \text{if } \lvert x\rvert < a \\ \mathrm{arcoth}\left(\frac{x}{a}\right) = \frac{1}{2}\log\left(\frac{x+a}{x-a}\right), & \text{if } \lvert x\rvert > a\end{cases}$
37	$\sqrt{a^2-x^2}$	$\frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\arcsin\left(\frac{x}{a}\right)$	38	$\sqrt{x^2-a^2}$	$\frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2\mathrm{arcosh}\left(\frac{x}{a}\right)$
39	$\sqrt{x^2+a^2}$	$\frac{1}{2}x\sqrt{x^2+a^2} + \frac{1}{2}a^2\mathrm{arsinh}\left(\frac{x}{a}\right)$	40	$\mathrm{e}^{kx}\sin(\omega x)$	$\frac{1}{k^2+\omega^2}\mathrm{e}^{kx}\left(k\sin(\omega x) - \omega\cos(\omega x)\right)$
41	$\mathrm{e}^{kx}\cos(\omega x)$	$\frac{1}{k^2+\omega^2}\mathrm{e}^{kx}\left(k\cos(\omega x) + \omega\sin(\omega x)\right)$	42	$\sin^2(\omega x)\cos^2(\omega x)$	$\frac{x}{8} - \frac{1}{32\omega}\sin(4\omega x)$